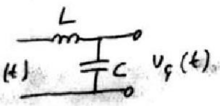
SİNYALLER ve SİSTEMLER BÜTÜNLEME SINAVI SORULAR

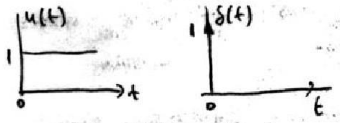
EMÜ442 HABERLEŞME TEKNİĞİ ARA SINAVI 21.04.98

S.1.) 

Yanda verilen devrenin, aşağıda verilen girişlere vereceği cevapların Fourier dönüşümlerini bulunuz. a.) $V_g(t)=\delta(t)$ b.) $V_g(t)=u(t)$ c.) $V_g(t)=u(t-T)$

a.) $V_g(t)=\delta(t) \Rightarrow V_q(t)=h(t)$, $F[h(t)]=H(j\omega)$, $H(j\omega)=H(s)\big|_{s=j\omega}$, $H(s)=\dfrac{\frac{1}{sC}}{sL+\frac{1}{sC}}=\dfrac{\frac{1}{LC}}{s^2+\frac{1}{LC}}$

$$H(j\omega)=\frac{1/LC}{-\omega^2+1/LC}$$

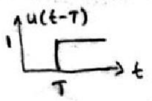
b.) 

$\dfrac{du(t)}{dt}=\delta(t)$, $u(t)=\int_{-\infty}^{t}\delta(z)\,dz$, $\int_{-\infty}^{t} f(z)\,dz \Leftrightarrow \frac{1}{j\omega}F(\omega)$

zaman integrali özelliği

$$V_g(t)=u(t) \Rightarrow F[V_q(t)] = \frac{1}{j\omega}\,\frac{1/LC}{-\omega^2+1/LC} = \frac{j\frac{1}{LC}}{\omega^3-\frac{\omega}{LC}}$$

c.) $V_g(t)=u(t-T)$ b.)deki giriş T kadar kaydırılmış. $f(t)\Leftrightarrow F(\omega)$, $f(t-t_0)\Leftrightarrow e^{-j\omega t_0}F(\omega)$ zaman kayma özelliği



$$F[V_q(t)] = e^{-j\omega T}\,\frac{j\frac{1}{LC}}{\omega^3-\frac{\omega}{LC}}$$

. (Her şık için de $|V_q(\omega)|$ leri çizmeye çalışınız)

C.4

$X(z)=\dfrac{z(2z+1)}{z^3+2z^2-z-2}$, YB : $|z|<1$ Uzun bölme kullanarak $X[n]=?$ bulunuz.

C.2)

$$
\begin{array}{r|l}
 & -\frac{1}{2}z-\frac{3}{4}z^2-\frac{1}{8}z^3-\frac{15}{16}z^4-\cdots \\
\hline
-2-z+2z^2+z^3 & z+2z^2 \\
 & -\; z\mp\frac{1}{2}z^2\pm z^3\pm\frac{1}{2}z^4 \\
\cline{2-2}
 & \frac{3}{2}z^2+z^3+\frac{1}{2}z^4 \\
 & -\;\frac{3}{2}z^2\mp\frac{3}{4}z^3\pm\frac{3}{4}z^5 \\
\cline{2-2}
 & \frac{1}{4}z^3+2z^4+\frac{3}{4}z^5 \\
 & -\frac{1}{4}z^3\mp\frac{1}{8}z^4\pm\frac{1}{4}z^5\pm\frac{1}{8}z^6 \\
\cline{2-2}
 & \frac{15}{8}z^4+z^5+\frac{1}{8}z^6 \\
 & -\;\frac{15}{8}z^4+\cdots \\
\cline{2-2}
 & \vdots
\end{array}
$$

$x[n]=0$ $n\geq 0$ için

$x[-1]=-1/2$

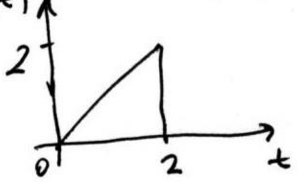
$x[-2]=-3/4$

$x[-3]=-1/8$

$x[-4]=-15/16$

$\vdots$

C.3.) $h(t) = t \quad 0 \le t \le 2$ $\qquad x(t) = \delta(t) - \delta(t-1)$



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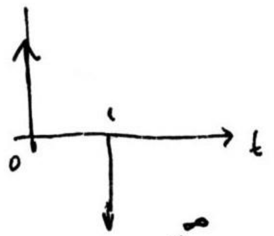
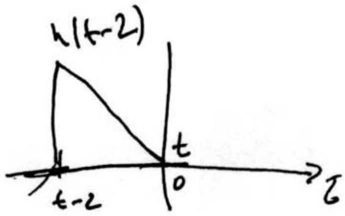
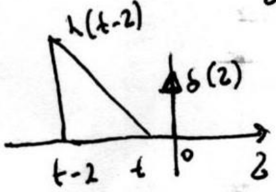




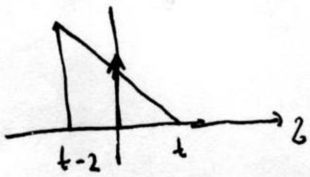
10p

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau = \int_{-\infty}^{\infty} [\delta(\tau) - \delta(\tau-1)][t-\tau]\, d\tau$$

$$y(t) = \underbrace{\int_{-\infty}^{\infty} \delta(\tau)(t-\tau)\, d\tau}_{y_1(t) \,\to\, \delta(t) * h(t)} - \underbrace{\int_{-\infty}^{\infty} \delta(\tau-1)(t-\tau)\, d\tau}_{y_2(t) \,\to\, \delta(t-1) * h(t)}$$

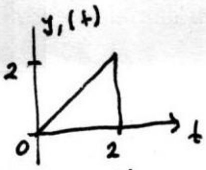


$t \le 0 \quad y_1(t) = 0$



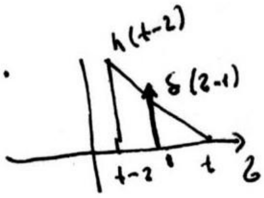
$$0 \le t \le 2 \quad y_1(t) = \int_{0^-}^{0^+} \delta(\tau)(t-\tau)\, d\tau = t\underbrace{\int_{0^-}^{0^+} \delta(\tau) d\tau}_{1} - \int_{0^-}^{0^+} \tau\, d\tau$$

$$y_1(t) = t - \frac{\tau^2}{2}\Big|_{0^-}^{0^+} = t - \left(\frac{1}{2} - \frac{1}{2}\right) = \underline{\underline{t}}$$

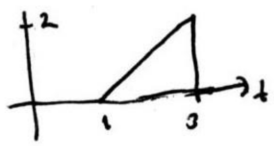


$$y_2(t) = \int_{-\infty}^{\infty} \delta(\tau-1)(t-\tau)\, d\tau = t\underbrace{\int_{1^-}^{1^+} \delta(\tau-1) d\tau}_{1} - \int_{1^-}^{1^+} \tau\, d\tau \qquad 1 \le t \quad y_2(t) = 0$$

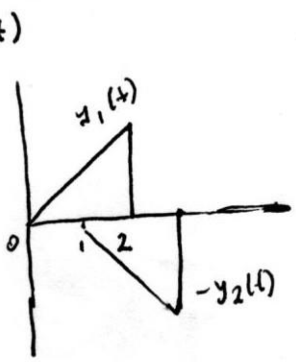
$$y_2(t) = t - \frac{\tau^2}{2}\Big|_{1^-}^{1^+} = t - \left(\frac{1}{2} - \frac{1}{2}\right) = \underline{\underline{t}} \qquad 1 \le t \le 3$$



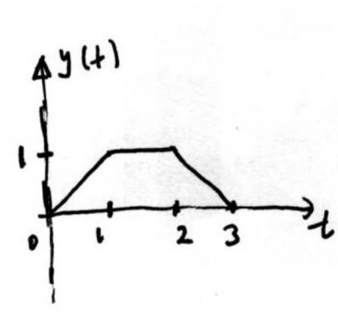
$y_2(t) =$



$y(t) = y_1(t) - y_2(t)$



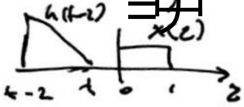
$\longrightarrow$



15p

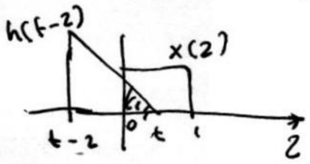
$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau)\, d\tau = \int_{-\infty} (t-\tau)\, d\tau = t\int d\tau - \int \tau\, d\tau$

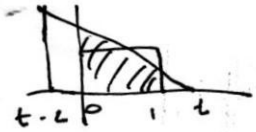
$y(t) = t\tau\Big|_0^1 - \frac{\tau^2}{2}\Big|_0^1$



$t \leq 0 \quad y(t) = 0$

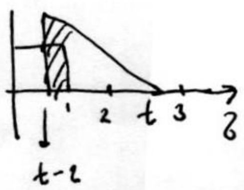
$0 \leq t \leq 1 \quad y(t) = \int_0^t (t-\tau)\, d\tau = t\tau\Big|_0^t - \frac{\tau^2}{2}\Big|_0^t = t^2 - \frac{t^2}{2} = \frac{t^2}{2}$





$1 \leq t \leq 2 \quad y(t) = \int_0^1 (t-\tau)\, d\tau$

$y(t) = t\tau\Big|_0^1 - \frac{\tau^2}{2}\Big|_0^1 = t - \frac{1}{2}$

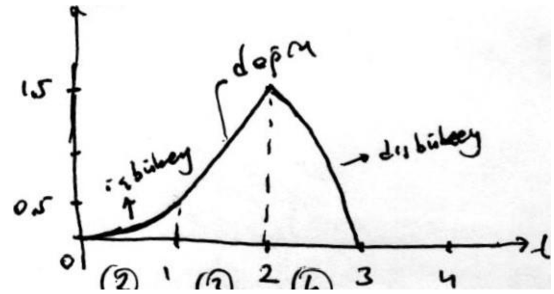


$2 \leq t \leq 3 \quad y(t) = \int_{t-2}^{1} (t-\tau)\, d\tau = t\tau\Big|_{t-2}^{1} - \frac{\tau^2}{2}\Big|_{t-2}^{1}$

$y(t) = t - t(t-2) - \frac{1}{2} + \frac{(t-2)^2}{2}$

$= t - t^2 + \cancel{2t} - \frac{1}{2} + \frac{t^2}{2} - \cancel{2t} + 2$

$y(t) = -\frac{t^2}{2} + t + \frac{3}{2}$ (2+)

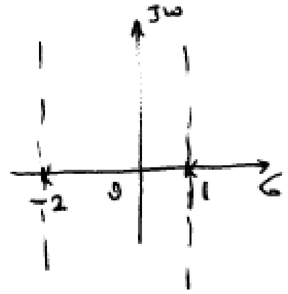


C.6.) $y''(t) + y'(t) - 2y(t) = x(t)$

a.) $s^2Y(s) + sY(s) - 2Y(s) = X(s)$

$(s^2+s-2)Y(s) = X(s)$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s^2+s-2} = \frac{1}{(s+2)(s-1)}$$



b.) Kısmi kesirlere açılımı kullanarak

$$H(s) = \frac{1}{(s+2)(s-1)} = -\frac{1}{3}\frac{1}{s+2} + \frac{1}{3}\frac{1}{s-1}$$

i.) Sistem nedensel ise h(t) nedenseldir yani sağ yanlıdır H(s)ın YB de sağ yanlı olacağından $Re(s)>1$ olmalıdır.

$$e^{-2t}u(t) \longleftrightarrow \frac{1}{s+2} \qquad Re(s) > -Re(2)$$

$$h(t) = -\frac{1}{3}e^{-2t}u(t) + \frac{1}{3}e^{t}u(t) = -\frac{1}{3}(e^{-2t} - e^{t})u(t) \quad //$$

ii.) Sistem kararlı ise YB jw eksenini içerir. YB -2 ile 1 arasındaki bölgedir yani h(t) çift yanlıdır.

$$e^{-2t}u(t) \longleftrightarrow \frac{1}{s+2} \qquad Re(s) > -Re(2)$$

YB

$$-\frac{1}{3}e^{-2t}u(t)$$

$$-e^{-2t}u(-t) \longleftrightarrow \frac{1}{s+2} \qquad Re(s) < -Re(2)$$

$$-\frac{1}{3}e^{t}u(-t)$$

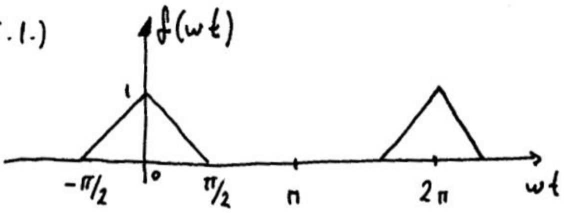
$$\boxed{h(t) = -\frac{1}{3}e^{-2t}u(t) - \frac{1}{3}e^{t}u(-t)}$$

iii.) Sistem nedensel değil, kararlı da değilse YB $Re(s) < -2$ dir.
(Yani nedensel değilse YB sol yanlı, kararlı değilse jw eks. içermeyecek)

$$-e^{-2t}u(-t) \longleftrightarrow \frac{1}{s+2} \qquad Re(s) < -Re(2)$$

$$\boxed{h(t) = \frac{1}{3}e^{-2t}u(-t) - \frac{1}{3}e^{t}u(-t)}$$

C.1.)



$$f(\omega t)=\begin{cases}1+\frac{2}{\pi}\omega t & -\frac{\pi}{2}<\omega t<0\\ 1-\frac{2}{\pi}\omega t & 0<\omega t<\frac{\pi}{2}\end{cases}$$

$$C_n=\frac{1}{2\pi}\int_{-\pi}^{\pi} f(\omega t)\, e^{-jn\omega t}\, d(\omega t)$$

$$C_n=\frac{1}{2\pi}\int_{-\pi/2}^{0}\left(1+\frac{2}{\pi}\omega t\right)e^{-jn\omega t}d(\omega t)+\frac{1}{2\pi}\int_{0}^{\pi/2}\left(1-\frac{2}{\pi}\omega t\right)e^{-jn\omega t}d(\omega t)$$

$$=\frac{1}{2\pi}\left\{\int_{-\pi/2}^{0}e^{-jn\omega t}d(\omega t)+\frac{2}{\pi}\int_{-\pi/2}^{0}\omega t\, e^{-jn\omega t}d(\omega t)+\int_{0}^{\pi/2}e^{-jn\omega t}d(\omega t)-\frac{2}{\pi}\int_{0}^{\pi/2}\omega t\, e^{-jn\omega t}d(\omega t)\right\}$$

$$=\frac{1}{2\pi}\left\{\int_{-\pi/2}^{\pi/2}e^{-jn\omega t}d(\omega t)+\frac{2}{\pi}\int_{-\pi/2}^{0}\omega t\, e^{-jn\omega t}d(\omega t)-\frac{2}{\pi}\int_{0}^{\pi/2}\omega t\, e^{-jn\omega t}d(\omega t)\right\}$$

$$C_n=\frac{1}{2\pi}\left\{\left[-\frac{1}{jn}e^{-jn\omega t}\right]_{-\pi/2}^{\pi/2}+\frac{2}{\pi}\left[e^{-jn\omega t}\left(\frac{1}{n^2}-\frac{\omega t}{jn}\right)\right]_{-\frac{\pi}{2}}^{0}-\frac{2}{\pi}\left[e^{-jn\omega t}\left(\frac{1}{n^2}-\frac{\omega t}{jn}\right)\right]_{0}^{\pi/2}\right\}$$

$\omega t=u$, $d\omega t=du$; $e^{-jn\omega t}d(\omega t)=dv$, $v=-\frac{1}{jn}e^{-jn\omega t}$

$$\int u\,dv=uv-\int v\,du$$
$$=-\omega t\frac{1}{jn}e^{-jn\omega t}-\int-\frac{1}{jn}e^{-jn\omega t}d(\omega t)$$
$$=-\omega t\frac{1}{jn}e^{-jn\omega t}+\frac{1}{n^2}e^{-jn\omega t}$$
$$=e^{-jn\omega t}\left(\frac{1}{n^2}-\frac{\omega t}{jn}\right)$$

$$C_n=\frac{1}{2\pi}\left\{-\frac{1}{jn}\left(e^{-jn\frac{\pi}{2}}-e^{jn\frac{\pi}{2}}\right)+\frac{2}{\pi}\left[\frac{1}{n^2}-e^{jn\frac{\pi}{2}}\left(\frac{1}{n^2}+\frac{\pi}{2jn}\right)\right]-\frac{2}{\pi}\left[e^{-jn\frac{\pi}{2}}\left(\frac{1}{n^2}-\frac{\pi}{2jn}\right)-\frac{1}{n^2}\right]\right\}$$

$$C_n=\frac{1}{2\pi}\left\{-\cancel{\frac{1}{jn}e^{-jn\frac{\pi}{2}}}+\cancel{\frac{1}{jn}e^{jn\frac{\pi}{2}}}+\frac{2}{\pi n^2}-\frac{2}{\pi n^2}e^{jn\frac{\pi}{2}}-\cancel{\frac{1}{jn}e^{jn\frac{\pi}{2}}}-\frac{2}{\pi n^2}e^{-jn\frac{\pi}{2}}+\cancel{\frac{1}{jn}e^{-jn\frac{\pi}{2}}}+\frac{2}{\pi n^2}\right.$$

$$C_n=\frac{2}{\pi^2n^2}-\frac{1}{\pi^2n^2}e^{jn\frac{\pi}{2}}-\frac{1}{\pi^2n^2}e^{-jn\frac{\pi}{2}}=\frac{2}{\pi^2n^2}-\frac{2}{\pi^2n^2}\left(\frac{e^{jn\frac{\pi}{2}}+e^{-jn\frac{\pi}{2}}}{2}\right)=\boxed{\frac{2}{\pi^2n^2}\left(1-\cos n\frac{\pi}{2}\right)}$$

ortalama değer $=\frac{\text{Alan}}{\text{peryot}}=\frac{1\cdot\frac{\pi}{2}}{2\pi}=\frac{1}{4}=0{,}25$

$$f(t)=\frac{1}{4}+\sum_{\substack{n=-\infty\\ n\neq 0}}^{\infty}\left(\frac{2}{\pi^2n^2}\left(1-\cos n\frac{\pi}{2}\right)\right)e^{jn\omega t}$$, $n=\ldots-2,-1,0,1,2\ldots$ için seri

$$f(t)=0.25+\frac{1}{2\pi^2}\left(1-\cos(-\pi)\right)e^{-j2\omega t}-\frac{2}{\pi^2}\left(1-\cos\left(-\frac{\pi}{2}\right)\right)e^{-j\omega t}+\frac{2}{\pi^2}\left(1-\cos\left(\frac{\pi}{2}\right)\right)e^{j\omega t}+\frac{1}{2\pi^2}\left(1-\cos(\pi)\right)e^{j2\omega t}$$

$$f(t)=0.25+0{,}101\,e^{-j2\omega t}+0{,}202\,e^{-j\omega t}+0{,}202\,e^{j\omega t}+0{,}101\,e^{j2\omega t}$$

dc bileşen 0,25 ; 1-harmonik $2*0{,}202\left(e^{-j\omega t}+e^{j\omega t}\right)\frac{1}{2}=0{,}404\cos\omega t$

2. harmonik $0{,}202\cos 2\omega t$