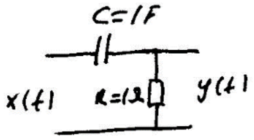
EMÜ311 SİNYALLER ve SİSTEMLER DERSİ ARASINAVI SORULARI 13.11.2016

S.1.)



a.) $H(s) = \frac{Y(s)}{X(s)} = ?$ bulunuz.

b.) Ters Laplace Dönüşümünü kullanarak $h(t) = ?$ bul, çiziniz.

c.) $s(t) = ?$ $\delta(t), u(t)$ ilişkisini kullanarak bulunuz, çiziniz.

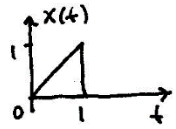
d.) $H(s)\big|_{s=j\omega} = H(j\omega)$ yı bulunuz. Sonuç doğrumudur? Neden?

S.2.) a.) $x(t) \leftrightarrow X(s)$, $x(t-t_0) \leftrightarrow ?$ L.D. tanım bağıntısında değişken değiştirerek bulunuz.

b.) $x(t) * \delta(t) = x(t)$ olduğunu L.D ve T.L.D kullanarak gösteriniz.

c.) $x(t) * \delta(t-t_0)$ L.Dönüşümünü bulunuz. T.L.D sonucunda bulduğunuz işareti herhangi bir $x(t)$ için çiziniz.

S.3.)



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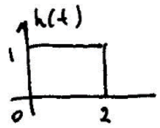
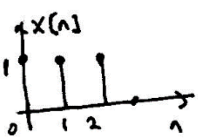


$= y(t) = \int_{-\infty}^{\infty} x(\tau)\,h(t-\tau)\,d\tau$ bulunuz, çiziniz.

S.4.)



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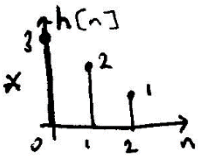


$= y[n] = \sum_{k=-\infty}^{\infty} x[k]\,h[n-k]$ bulunuz. çiziniz. a.)

b.) Konvolusyon kullanmadan $y[n] = ?$

Sınav süresi 90 dakikadır. Başarılar dilerim.

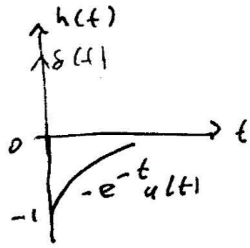
Melih Cevdet İnce

C.1.) a.) $H(s) = \frac{1}{1+\frac{1}{s}} = \frac{1}{\frac{s+1}{s}} = \frac{s}{s+1}$

b.) Kısmi kesirlere ayılım için $P(s)^\circ < Q(s)^\circ$ olmalı. değil. Pay paydaya bölersek

$H(s) = 1 - \frac{1}{s+1} \Rightarrow h(t) = \delta(t) - e^{-t}.u(t)$

$Re(s) > -1$



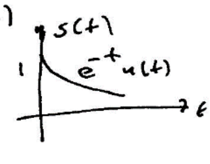
c.) $s(t) = ?$ $u(t) = \int_{-\infty}^{t} \delta(\tau)\,d\tau$

$s(t) = \int_{-\infty}^{t} h(\tau)\,d\tau = \int_{-\infty}^{t} \delta(\tau) - e^{-\tau}u(\tau)\,d\tau = \int_{-\infty}^{t} \delta(\tau)\,d\tau - \int_{-\infty}^{t} e^{-\tau}u(\tau)\,d\tau$

$= u(t) - \left[-e^{-\tau}\right]_0^t = u(t) - \left[-e^{-t}+1\right]$

$= u(t) + e^{-t} - 1$

$s(t) = e^{-t}.u(t)$



d.) $H(s)\big|_{s=j\omega} = \dfrac{j\omega}{j\omega+1}$

sonuç doğrudur. Çünkü $\int_{-\infty}^{\infty} |h(t)|\, dt < \infty$ Açıklama /

$\int_{-\infty}^{\infty} |\delta(t) - e^{-t}.u(t)|\, dt$, $\int_{-\infty}^{\infty} |\delta(t)|\, dt = 1$

$\int_{0}^{\infty} |-e^{-t}|\, dt = e^{-t}\Big|_{0}^{\infty} = 0-1=-1$

$\int_{-\infty}^{\infty} |\delta(t) - e^{-t}u(t)|\, dt = 0 < \infty$

C.2.a.) $\int_{-\infty}^{\infty} x(t-t_0)\, e^{-st}\, dt = \int_{-\infty}^{\infty} x(z)\, e^{-s(z+t_0)}\, dz = e^{-st_0}\int_{-\infty}^{\infty} x(z)\, e^{-sz}\, dz$

$z = t - t_0$

$t = z + t_0$

$dt = dz$

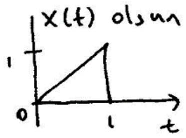
$\mathcal{L}\{x(t-t_0)\} = e^{-st_0} X(s)$

b.) $x(t) * \delta(t) \longleftrightarrow X(s).1$, $\mathcal{L}^{-1}\{X(s)\} = x(t)$

c.) $x(t) * \delta(t-t_0) \Longleftrightarrow X(s).e^{-st_0}$

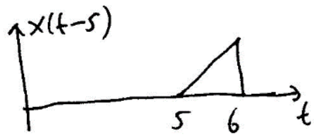
a-) şıkkından $x(t-t_0) \longleftrightarrow e^{-st_0}X(s)$ olduğu gösterildi. o halde,

$\mathcal{L}^{-1}\{X(s)e^{-st_0}\} = x(t-t_0)$

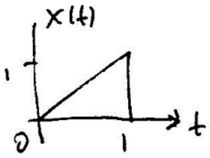


$x(t) = \begin{cases} t & 0 \le t \le 1 \\ 0 & d.y. \end{cases}$ $\quad x(t-5) = \begin{cases} t-5 & 0 \le t-5 \le 1, \ 5 \le t \le 6 \\ 0 & \text{diğer yerlerde.} \end{cases}$

$t_0 = 5$ olsun.



C-3.)



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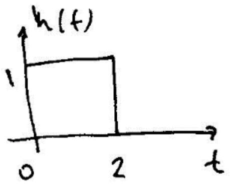


$= y(t) = \int_{-\infty}^{\infty} x(\tau)\,h(t-\tau)\,d\tau$

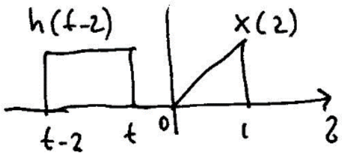
$$h(t) = \begin{cases} 1 & 0 \leq t \leq 2 \\ 0 & \text{d.y.} \end{cases}$$

$$h(t-\tau) = \begin{cases} 1 & 0 \leq t-\tau \leq 2 \\ 0 & \text{d.y} \end{cases}$$

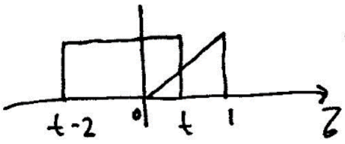
$0 - t \leq -\tau \leq 2 - t$

$t \geq \tau \geq t-2$

$\underline{\underline{t-2 \leq \tau \leq t}}$

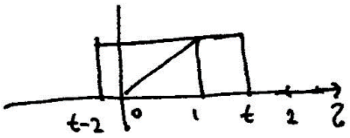


$t \leq 0 \Rightarrow y(t) = 0$ (1)



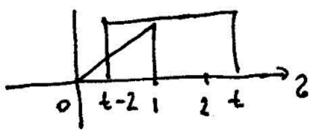
$0 \leq t \leq 1 \Rightarrow y(t) = \int_0^t \tau \, d\tau$

$y(t) = \frac{\tau^2}{2} \Big|_0^t = \frac{t^2}{2}$ (2)

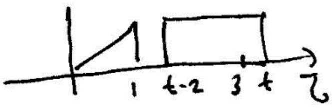


$1 \leq t \leq 2 \Rightarrow y(t) = \frac{\tau^2}{2} \Big|_0^1$

$y(t) = \frac{1}{2}$ (3)

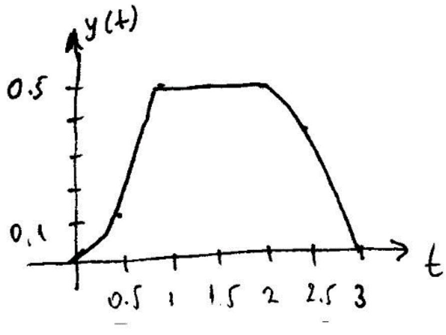


$2 \leq t \leq 3 \Rightarrow y(t) = \frac{\tau^2}{2} \Big|_{t-2}^{1} = \frac{1}{2} - \frac{(t-2)^2}{2}$ (4)

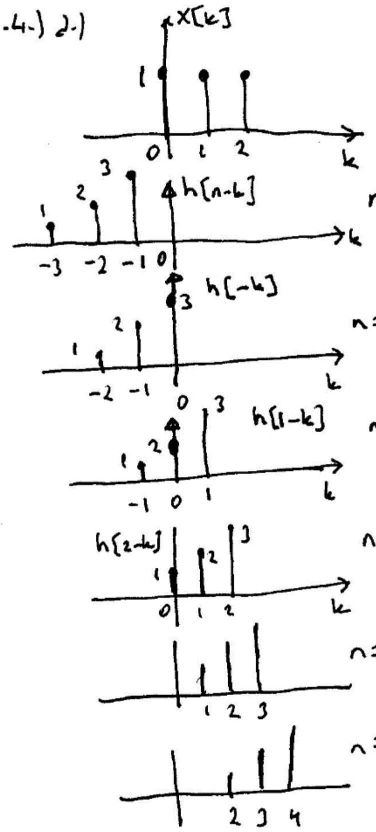


$t \leq 3 \Rightarrow y(t) = 0$ (5)

t	$y(t)$
0	0 (1),(2)
0,5	0,125 (2)
1	$\frac{1}{2}$ (2),(3)
1,5	$\frac{1}{2}$ (3)
2	$\frac{1}{2}$ (3),(4)
2,5	0,375 (5)
3	0 (4),(5)



C.4.) a.)



$$y[n]=\sum_{k=-\infty}^{\infty} x[k]\,h[n-k]$$

$n=-1 \Rightarrow y[n]=\sum_{k=-\infty}^{\infty} x[k]h[n-k] = y[-1]=0$

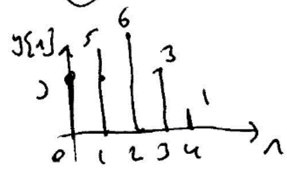
$n=0 \Rightarrow y[0]=\sum_{k=0}^{0} x[0]h[0] = 1\cdot 3 = 3$

$n=1 \Rightarrow y[1]=\sum_{k=0}^{1} x[0]h[0]+x[1]h[1] = 1\cdot 2+1\cdot 3 = 2+3 = 5$

$n=2 \Rightarrow y[2]=\sum_{k=0}^{2} x[0]h[0]+x[1]h[1]+x[2]h[2] = 1+2+3 = 6$

$n=3 \Rightarrow y[3]=\sum_{k=1}^{2} x[1]h[1]+x[2]h[2] = 1+2 = 3$

$n=4 \Rightarrow y[4]=\sum_{k=2}^{2} x[2]h[2] = 1\cdot 1 = 1$



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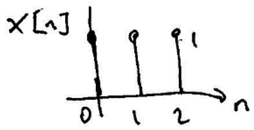
b.)



$x[n]=\delta[n]+\delta[n-1]+\delta[n-2]$

$y[n]=h[n]+h[n-1]+h[n-2]$